MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

DEM5018 – ENGINEERING MATHEMATICS 1

(DEE)

1 JUNE 2017
2.30 p.m - 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **8** pages (**5** pages with **6** questions and **3** pages for appendix)

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

QUESTION 1 [10 Marks]

a) Refer to Figure 1, state the solution set in interval notation. [1 mark]

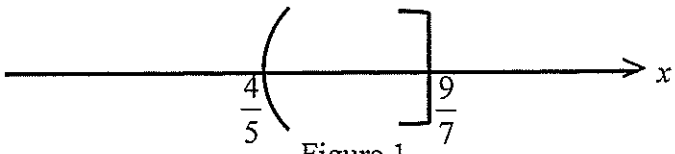

Figure 1

b) State whether the circle with the equation, $x^2 - 5x + y^2 - y + 5 = 0$, passes through y-axis. [3 marks]

c) Given that the conditions of $\csc\theta < 0$ and $\cot\theta > 0$, name the quadrant in which the angle θ lies, if θ is $0 < \theta < 90°$. [1 mark]

d) State the conjugate of $111 - \sqrt{11}i$. [1 mark]

e) Referring to the graph in Figure 2, identify whether $f(x)$ and $g(x)$ is either odd function or even function. [2 marks]

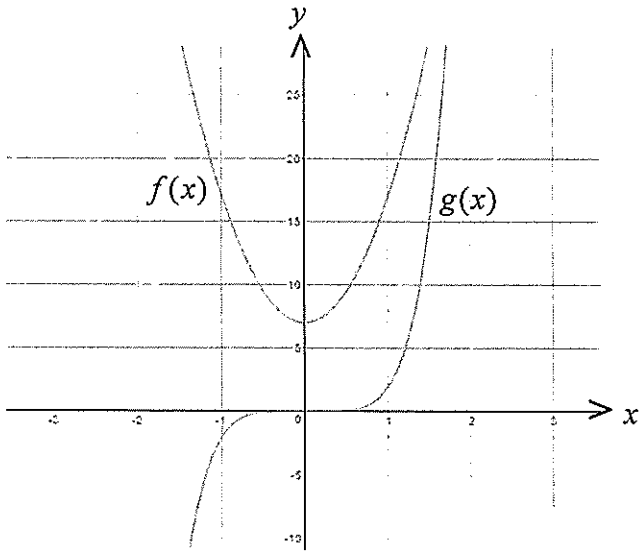

Figure 2

Continued...

f) Referring to the graph in Figure 3, state whether the following limits exist or does not exist.

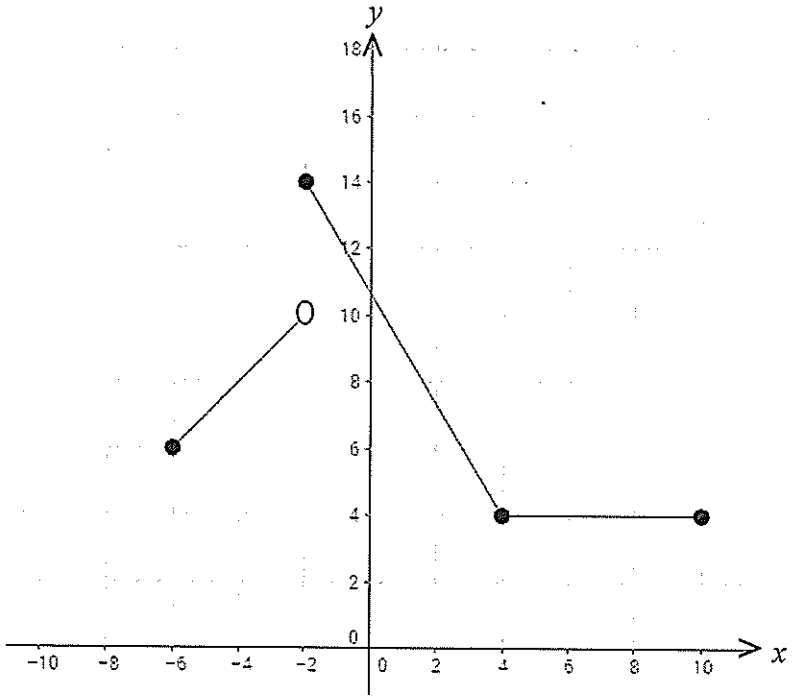

Figure 3

(i) $\lim_{x \to -2} f(x)$

(ii) $\lim_{x \to 4} f(x)$ [2 marks]

Continued...

QUESTION 2 [20 Marks]

a) Referring to Figure 1, answer the following questions.

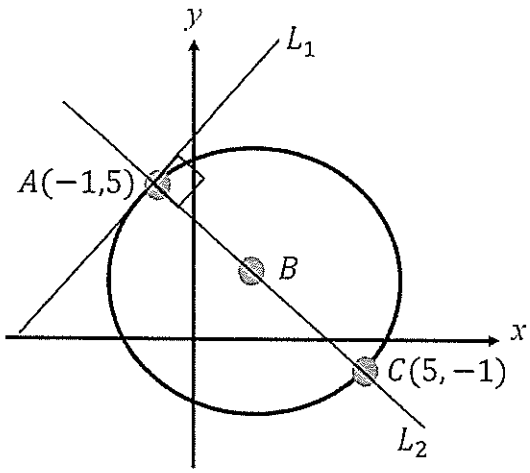

Figure 1

(i) Given B is the center of the circle, determine the coordinate of B and radius of the circle. Then, state the circle equation for the circle, in general form. [6 marks]

(ii) Given L_1 and L_2 are perpendicular to each other, find the linear equation of L_1. [4 marks]

b) Solve the inequality $\frac{2x-3}{x+5} \geq 0$, where $x \neq -5$ and illustrate your answer on a number line. [6 marks]

c) Solve the inequality $\left|\frac{x}{4}-3\right|-11<5$. [4 marks]

Continued...

QUESTION 3 [20 Marks]

a) Given $\tan\alpha = \frac{7}{24}$, where $0 < \alpha < \frac{\pi}{2}$ and $\sec\beta = -\frac{5}{4}$, where $\pi < \beta < \frac{3\pi}{2}$, find the exact value of

(i) $\sin(\alpha - \beta)$ [4 marks]

(ii) $\tan(2\beta) - 1$ [4 marks]

b) Given $p = 1 + 2i$ and $q = -1 + i$,

(i) represent p in polar form. [3 marks]

(ii) determine $\overline{p+q}$ in standard form. [2 marks]

c) Given $w = 4(\cos 200° + i\sin 200°)$ and $p = \frac{1}{2}(\cos 245° + i\sin 245°)$,

(i) solve $\frac{w}{p}$, and leave your answer in rectangular form. [4 marks]

(ii) determine w^4, and leave your answer in polar form. [3 marks]

QUESTION 4 [20 Marks]

a) Find $f^{-1}(x)$ for $f(x) = \frac{2x-1}{3-x}$ where $x \neq 3$. Then evaluate $f^{-1}(-1)$. [6 marks]

b) Solve $\frac{dy}{dx}$ for $x^2 y = 5\sin x + 2e^y$ by using implicit differentiation. [6 marks]

c) Given $y = \frac{1}{8}\tan^{-1}(x^4)$, find the first derivative. [4 marks]

d) Given $g(x) = \ln e^{x^3-1}$, evaluate $g''(-2)$. [4 marks]

Continued...

QUESTION 5 [15 Marks]

Given a polynomial function, $y = \frac{1}{5}x^5 - \frac{4}{3}x^3 + 9$,

a) find the critical point(s) of the function. [7 marks]

b) identify the maximum and minimum point of the function. [5 marks]

c) find the intervals of concavity of the function if the inflection points are at $-\sqrt{2}$, 0 and $\sqrt{2}$. [3 marks]

QUESTION 6 [15 Marks]

a) Show that the limit of $\lim_{x\to\infty} \frac{10x^5 + 3x^2 - 1}{5x^5 - 7x^3}$ is 2. [5 marks]

b) Prove that $\frac{\cot x}{1 - \sec^2 x} = -\cot^3 x$. [5 marks]

c) Show that solution of the equation, $x^2 - 2x + 10 = 0$ are $1 + 3i$ and $1 - 3i$. [5 marks]

End of Page.

APPENDIX

A Standard Form of Circle Equation

$$(x-h)^2+(y-k)^2=r^2$$

B Fundamental Identities

- Even & Odd Properties: $\sin(-\theta)=-\sin\theta$ $\quad \csc(-\theta)=-\csc\theta$
  $\cos(-\theta)=\cos\theta$ $\quad \sec(-\theta)=\sec\theta$
  $\tan(-\theta)=-\tan\theta$ $\quad \cot(-\theta)=-\cot\theta$
- Reciprocal identities: $\csc\theta=\dfrac{1}{\sin\theta}$, $\sec\theta=\dfrac{1}{\cos\theta}$, $\cot\theta=\dfrac{1}{\tan\theta}$
- Pythagorean identities : $\sin^2\theta+\cos^2\theta=1$, $\quad \tan^2\theta+1=\sec^2\theta$,
  $1+\cot^2\theta=\csc^2\theta$
- Sum/Difference Formula for Cosine:
  $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$
- Sum/Difference Formula for Sine:
  $\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$
- Sum/Difference Formula for Tangent:
  $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$
- Double Angle Formula: $\cos 2A=\cos^2 A-\sin^2 A$
  $=1-2\sin^2 A$
  $=2\cos^2 A-1$
  $\sin 2A=2\sin A\cos A$
  $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$
- Half Angle Formula: $\sin\left(\dfrac{\alpha}{2}\right)=\pm\sqrt{\dfrac{1-\cos\alpha}{2}}$
  $\cos\left(\dfrac{\alpha}{2}\right)=\pm\sqrt{\dfrac{1+\cos\alpha}{2}}$
  $\tan\left(\dfrac{\alpha}{2}\right)=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

C Relations between Polar and Rectangular Coordinates

$x = r\cos\theta$; $y = r\sin\theta$; $r = \sqrt{x^2 + y^2}$; $\tan\theta = \frac{y}{x}$

D Products and Quotients in Polar Form

- $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]; \quad z_2 \neq 0$

E DeMoivre's Theorem

- Powers of Complex Numbers on Polar Form:
  $z^n = r^n(\cos n\theta + i\sin n\theta)$

- Roots of Complex Numbers in Polar Form:
  $z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right]$
  $k = 0, 1, 2, \ldots, n-1$

F Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

G The Chain Rule

If $y = f[g(x)]$ and $u = g(x)$, then $y = f(u)$:

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ *or* $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$

H Derivative of the Exponential & logarithmic Function

$$\frac{d}{dx}[e^x] = e^x; \qquad \frac{d}{dx}[e^u] = e^u.\frac{du}{dx}; \qquad \frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0 \quad ; \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}.\frac{du}{dx};$$

$$\frac{d}{dx}\log_a x = \frac{1}{\ln a}\cdot\frac{1}{x} = \frac{1}{x\ln a}$$

I Derivative of Inverse Trigonometry Function

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\csc^{-1} x\right] = -\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = -\frac{1}{1+x^2}$$

J Derivative of Hyperbolic Function

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x \qquad \frac{d}{dx}[\tanh x] = \sec h^2 x$$

$$\frac{d}{dx}[\sec hx] = -\sec hx \tanh x \qquad \frac{d}{dx}[\coth x] = -\csc h^2 x$$

$$\frac{d}{dx}[\csc hx] = -\csc hx \coth x$$

K The Second Derivative Test

Suppose f'' is continuous near c.

If $f'(c) = 0$ and $f''(c) > 0$, then f has local minimum at c.

If $f'(c) = 0$ and $f''(c) < 0$, then f has local maximum at c.